Remove Useless Variables and Productions in a CFG
Let C be a CFG (V, T, S, P)
Let F be a set of variables (nonterminals)
Let Q be a FIFO Queue
for each production A -> w
    if A does not belong to F and w belongs to T*
        add A to F
        insert A into Q
while Q is not empty
    Let B be the first value in Q
    Remove B from Q
    for each production A -> XBY where X and Y belong to (V union T)*
        if A does not belong to Q and every variable in X and Y belong to F
            add A to F
            insert A into Q
Remove all productions from C where the left hand side is not in F or the right hand side contains a variable not in F.

Remove variables from C that do not appear in any of the remaining productions.

Build a directed graph from the remaining variables. Add an edge (A, B) to the graph if there is a production in the graph where A is on the left hand side of the production and B appears in the right hand side of the production.

Start a breadth first search of the graph beginning at node S. Mark the nodes visited in the breadth first search.

Remove all productions that include a node that has not been marked.

Remove variables from C that do not appear in any of the remaining productions.