Generalized Nondeterministic Finite Automata
Definition of a Generalized Nondeterministic finite automata (GNFA)

- A generalized nondeterministic finite automaton is a 5 tuple $(Q, \Sigma, \delta, q_{\text{start}}, q_{\text{accept}})$ where
  - 1. $Q$ is a finite set of states
  - 2. $\Sigma$ is a finite set of symbols called an alphabet
  - 3. $\delta: (Q-\{q_{\text{accept}}\}) \times (Q-\{q_{\text{start}}\}) \rightarrow R$ is the transition function.
     - $R$ is the set of regular expressions over $\Sigma$.
  - 4. $q_{\text{start}}$ is the start state
  - 5. $q_{\text{accept}}$ is the final or accept states
GNFA Structure

• The start state has a transition to every other state.
• No transitions to the start state
• There is only a single accept state and it is not the start state
• All states have a transition to the accept state
• No transitions leave the accept state
• Except for the start state and the accept state every state has a transition to itself and to every other state.
Convert DFA to GNFA

• Add a new start state with an epsilon transition to the old start state

• Add a new accept state. All old accept states have an epsilon transition to the new accept state

• If a transition has multiple labels or there are multiple transitions from state q to state p (q and p could be the same state) replace the transition(s) with a single transition labeled with the union of the labels on the original transition(s)

• Add transitions labeled with the empty set between states (and between a state and itself) that do not have any transitions between them. This step is include so the transition function has a value for each pair of inputs. You can ignore these transitions when you do the conversion.
Reduce the GNFA to a 2 State GNFA

Let $G_1$ be a GNFA $(Q_1, \Sigma, \delta_1, q_{\text{start}}, q_{\text{accept}})$
Let $G_2$ be a GNFA $(Q_2, \Sigma, \delta_2, q_{\text{start}}, q_{\text{accept}})$

While the size of $Q_1 > 2$
  
  choose a state $q$ that is not $q_{\text{start}}$ or $q_{\text{accept}}$

  $Q_2 = Q_1 - \{q\}$

  for each $q_i$ in $Q_2 - \{q_{\text{accept}}\}$ and $q_j$ in $Q_2 - \{q_{\text{start}}\}$

    let $\delta_2(q_i, q_j) = (R1)(R2)^*(R3)U(R4)$

    where $R1 = \delta_1(q_i, q)$, $R2 = \delta_1(q, q)$,
    $R3 = \delta_1(q, q_i)$ and $R4 = \delta_1(q_i, q_i)$

$G_1 = G_2$