CS442/542

DFA Minimization
DFA Minimization

- Minimize the DFA \( (D, \Sigma, \delta, d_0, D_A) \)
- The algorithm builds a new machine from subsets of states of the original machine
- The algorithm first builds two subsets: the set of final states and the set of non-final states
- A subset is split if the subset has a conflict on a symbol
- A subset has a conflict on a symbol, \( c \), when the transitions on \( c \) of two (or more) states in the subset do not go to states in the same subset.
- The algorithm halts when no subsets have conflicts (i.e. no more splits need to be done)
Split (S is a set of states from the original DFA)

Split(S) {
    for each \( c \in \Sigma \) do
        if \( c \) splits \( S \) into \( s_1 \) and \( s_2 \) then return \( \{s_1, s_2\} \);
    end;
    return \( S \);
}


DFA Minimization

\[ T = \{ D_A, (D - D_A) \}; \]

\[ P = \emptyset \]

while ( \( P \neq T \) ) do

\[ P \leftarrow T; \]

\[ T \leftarrow \emptyset; \]

for each \( p \in P \) do

\[ T = T \cup \text{Split}(p); \]

end;

end;
DFA for \((0^*10^*10^*)^*\)
DFA Minimization

• Round 1
  
<table>
<thead>
<tr>
<th></th>
<th>S0, S4, S5</th>
<th>S1, S2, S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>S1, S5, S5</td>
<td>S1, S3, S3</td>
</tr>
</tbody>
</table>

• Round 2
  
<table>
<thead>
<tr>
<th></th>
<th>S0</th>
<th>S4, S5</th>
<th>S1, S2, S3</th>
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<tbody>
<tr>
<td>1</td>
<td>S2</td>
<td>S2, S2</td>
<td>S2, S4, S4</td>
</tr>
</tbody>
</table>

• Round 3
  
<table>
<thead>
<tr>
<th></th>
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<th>S1</th>
<th>S2, S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>S1</td>
<td>S5, S5</td>
<td>S1</td>
<td>S3, S3</td>
</tr>
<tr>
<td>1</td>
<td>S2</td>
<td>S2, S2</td>
<td>S2</td>
<td>S4, S4</td>
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DFA Minimization

The diagram shows a DFA with states labeled $S_0$, $S_1$, $S_2$, $S_3$, $S_4$, and $S_5$. The transitions are labeled with 0s and 1s, indicating the input symbols that lead to the next state.
DFA for $0^*(10^*10^*)^*$
## DFA Minimization

- **Round 1**

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<td>S3, S3</td>
</tr>
<tr>
<td>1</td>
<td>S2, S2, S2, S2</td>
<td>S4, S4</td>
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DFA Minimization

State Transition Diagram:
- Initial State: S0
- Final State: S1
- Transitions:
  - From S0 on 0 to S1
  - From S1 on 0 to S0
  - From S0 on 1 to S0
  - From S1 on 1 to S1
DFA Minimization

• Potential Problems
  – The transition function is not total
  – Dead states