Sorting Analysis
Average number of comparisons to insert a value into a sorted array containing \( n \) values:

\[
\frac{1}{n+1} \sum_{j=1}^{n+1} \left( \frac{j(j+1)}{2} \right) = \frac{n+2}{2}
\]
Average number of comparisons to sort an array of $n$ values, using an insertion sort:

\[\sum_{l=1}^{n-1} \frac{l+2}{2} = \frac{1}{2} \sum_{l=1}^{n-1} l + 2 = \frac{1}{2} \left( \sum_{l=1}^{n-1} l + \sum_{l=1}^{n-1} 2 \right)\]

\[= \frac{1}{2} \left( \frac{n(n-1)}{2} + 2(n-1) \right) = \frac{n(n-1)}{4} + (n-1)\]
Merge Sort Analysis (n is a power of 2)

**Recurrence Relation for Merge Sort**

\[ T(n) = 2T(n/2) + n \]

**How to Solve?**

Restructure the equation so terms cancel

Divide by sides by \( n \)

\[ \frac{T(n)}{n} = \frac{2T(n/2)}{n} + \frac{1}{n} \]

\[ \frac{T(n)}{n} = \frac{T(n/2)}{n} + \frac{1}{n/2} \]
Merge Sort Analysis (n is a power of 2)

Let's assume $n$ is a power of 2.

\[
\begin{align*}
\frac{T(n)}{n} &= \frac{T(n/2)}{n/2} + 1 \\
\frac{T(n/2)}{n/2} &= \frac{T(n/4)}{n/4} + 1 \\
\frac{T(n/4)}{n/4} &= \frac{T(n/8)}{n/8} + 1 \\
\vdots \\
\frac{T(2)}{2} &= \frac{T(1)}{1} + 1
\end{align*}
\]
Merge Sort Analysis (n is a power of 2)

Set the sum of the LHS equal to the sum of the RHS and cancel common terms.

Since $n$ is a power of 2, there are $\log_2 n$ equations, and thus $\log_2 n$ of 1s.

\[
\frac{T(n)}{n} = T(1) + \log_2 n
\]

\[
T(n) = n T(1) + n \log_2 n = n + n \log_2 n
\]
QuickSort Analysis

Recurrence relation

\[ T(0) = T(1) = 1 \]

\[ T(N) = T(i) + T(N-i-1) + N, \text{ for } N > 1 \]
QuickSort Analysis

Worst Case Analysis

\[ T(N) = T(N-1) + N, \text{ for } N > 1 \]

Telescope the equation

\[ T(N) = T(N-1) + N \]
\[ T(N-1) = T(N-2) + (N-1) \]
\[ T(N-2) = T(N-3) + (N-2) \]
\[ \ldots \]
\[ T(2) = T(1) + 2 \]

Sum the equations and cancel matching terms in the left and right resulting

\[ T(N) = T(1) + N + (N-1) + \ldots + 2 \]
QuickSort Analysis

Best Case

\[ T(N) = 2T(N/2) + N, \text{ for } N > 1 \]

This can be solved the same way we solved the recurrence relation for merge sort
QuickSort Analysis

Average case

Average the recurrence equation for all possible values of $i$ (all possible locations of the pivot element)

$$\frac{1}{N} \sum (T(i) + T(N-i-1)) + N$$

See textbook for a solution