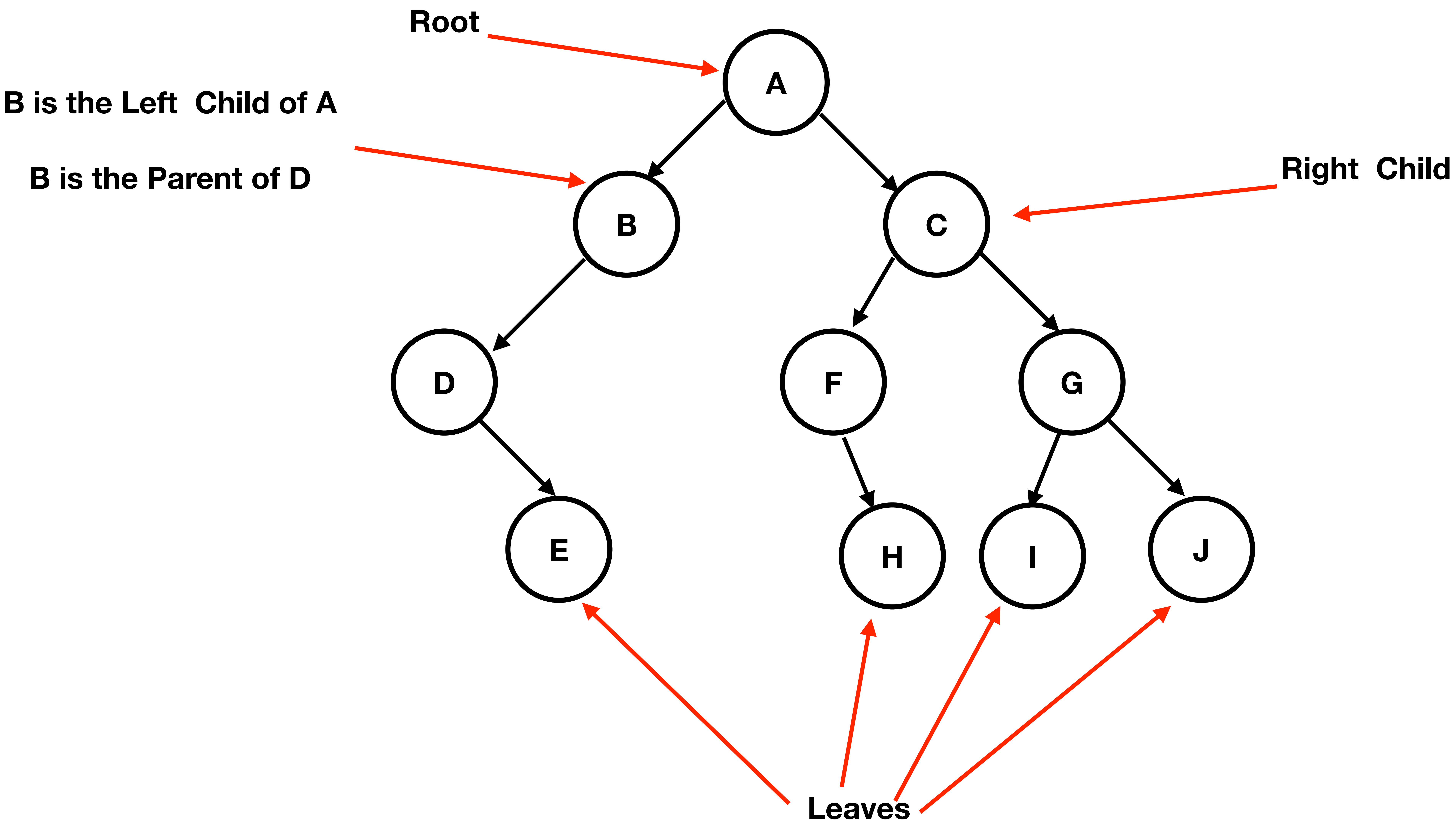
Binary Trees

Read Sections 4.1 and 4.2

Binary Trees



Binary Tree

A **tree** is a collection of nodes. A tree is either empty or it contains a node called the **root** that is linked to zero or more subtrees. The subtrees of a node T are called the **children** of T and T is called the **parent** of the subtrees.

A **binary tree** is a tree where each node has at most 2 children.

A node with zero children is called a **leaf.**

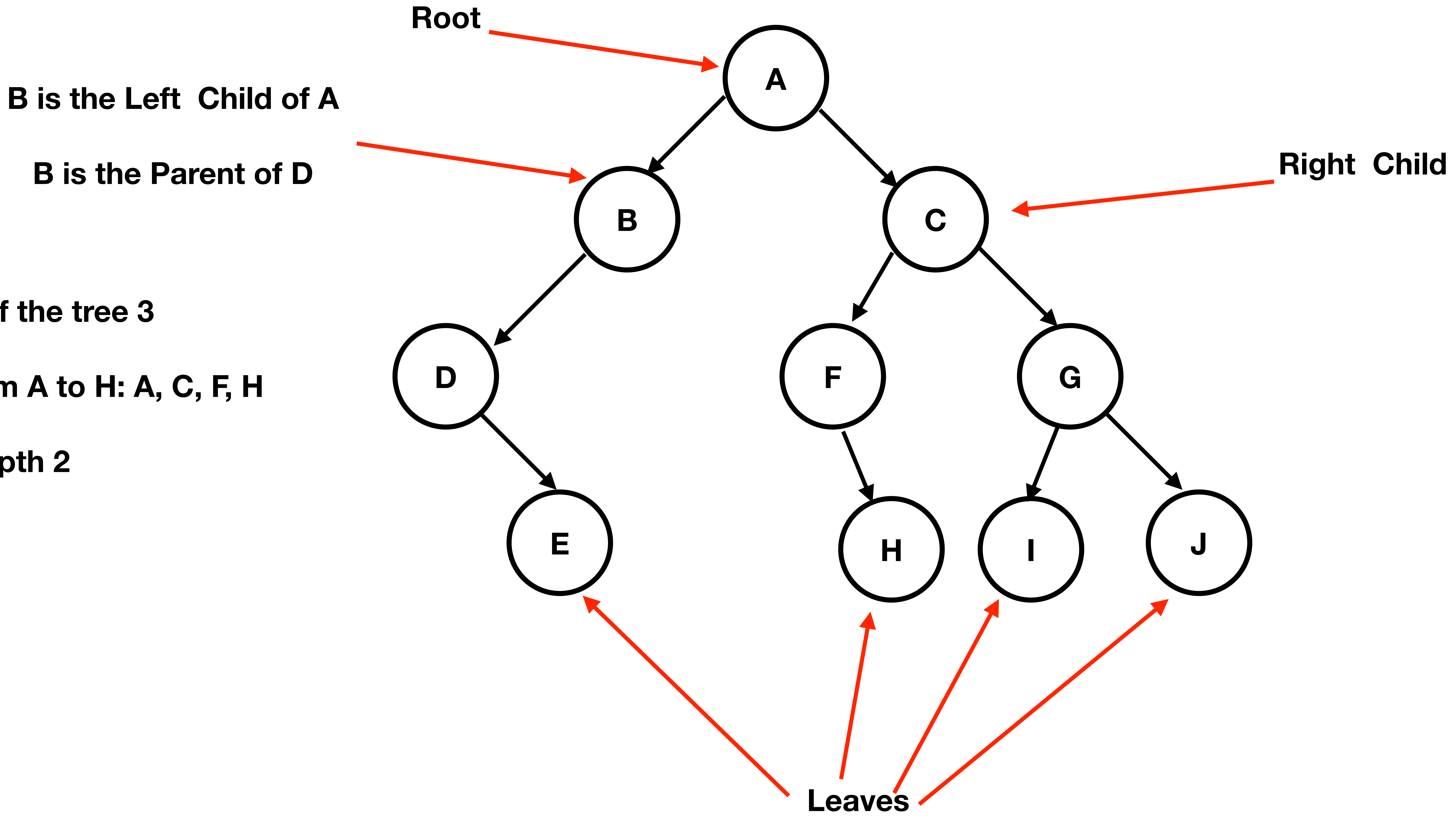
A **path** in a tree is a sequence of nodes, n_0, n_1, ... n_j, where n_i is the parent of n_{i+1} for $0 <= i < j$

The **length** of a path is one less than the number of nodes in the path (this is equivalent to saying the length is the number of edges in the path).

The **height** of a tree is the length of the longest path from the root to a leaf. The height of an empty tree is -1. The height of a tree with only one node (the root) is 0.

The **depth** of a node, n, is the length of the path from the root to n.

Binary Trees

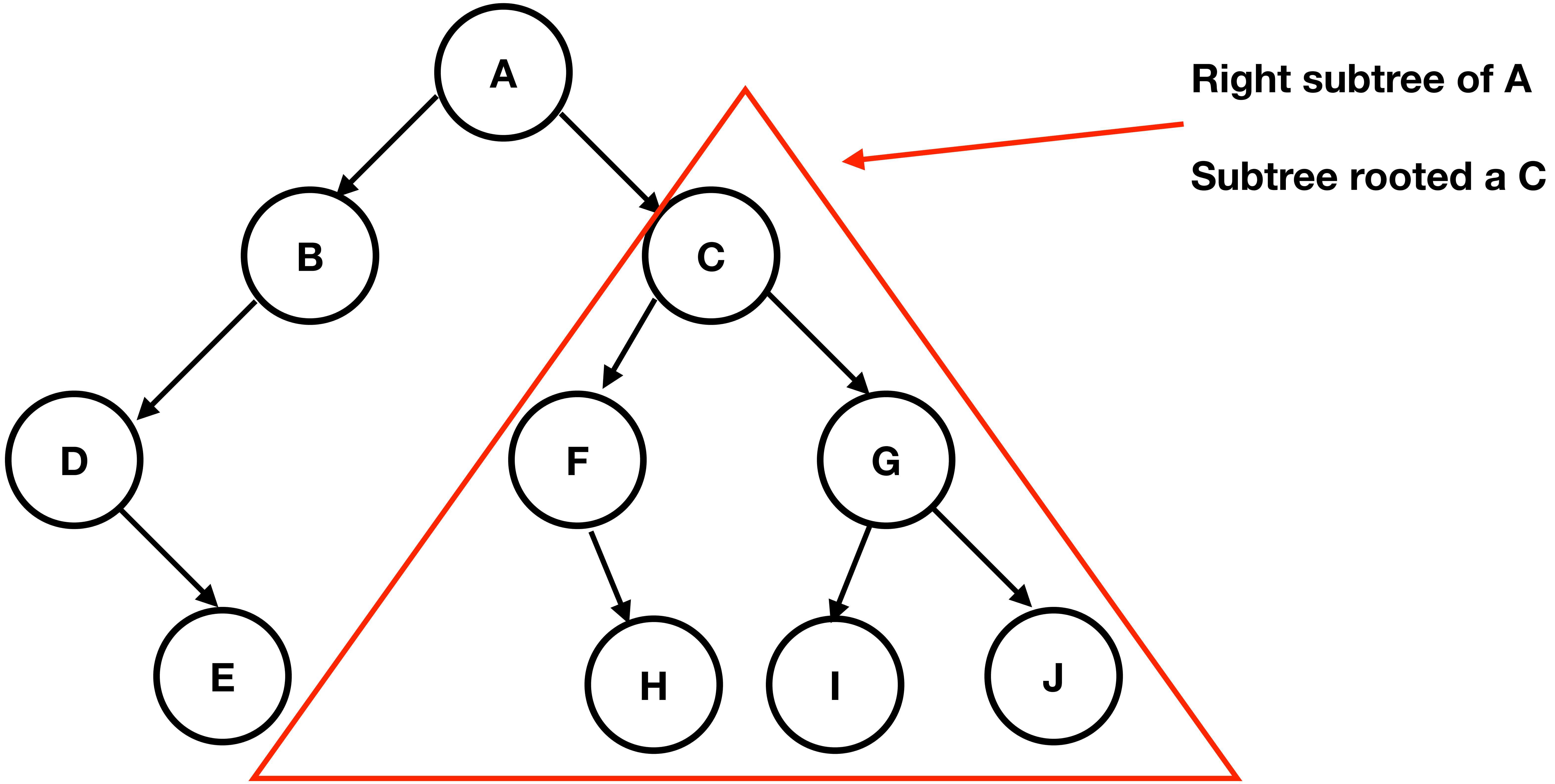


Height of the tree 3

Path from A to H: A, C, F, H

F is a depth 2

Binary Trees



Binary Trees

A **preorder** traversal of a binary tree rooted at node n, is a traversal where node n is visited (the meaning of visited will vary based on the purpose of the traversal) followed by a preorder traversal of the left subtree of n followed by a preorder traversal of the right subtree of n

A **postorder t**raversal of a binary tree rooted at node n, is a traversal where a postorder traversal of the left subtree of n is completed followed by a postorder traversal of the right subtree of n followed by a visit to n.

An **inorder** traversal of a binary tree rooted at node n, is a traversal where an inorder traversal of the left subtree of n is completed followed by a visit to n followed by an inorder traversal of the right subtree of n

A **level** order traversal of a tree rooted at node n, is a traversal where the nodes in the tree a visited based on the depth of the node: n is visited, followed by all nodes at depth 1, followed by all nodes at depth 2, ... until all the nodes at a depth equal to the height of the tree have been visited. A level order traversal is sometimes called a **breadth first** traversal.

Binary Tree

Preorder traversal

If the tree is not empty
- **visit(root)**
- **preOrder(left subtree)**
- **preOrder(right subtree)**

Inorder traversal

If the tree is not empty
- **inOrder(left subtree)**
- **visit(root)**
- **inOrder(right subtree)**

Postorder traversal

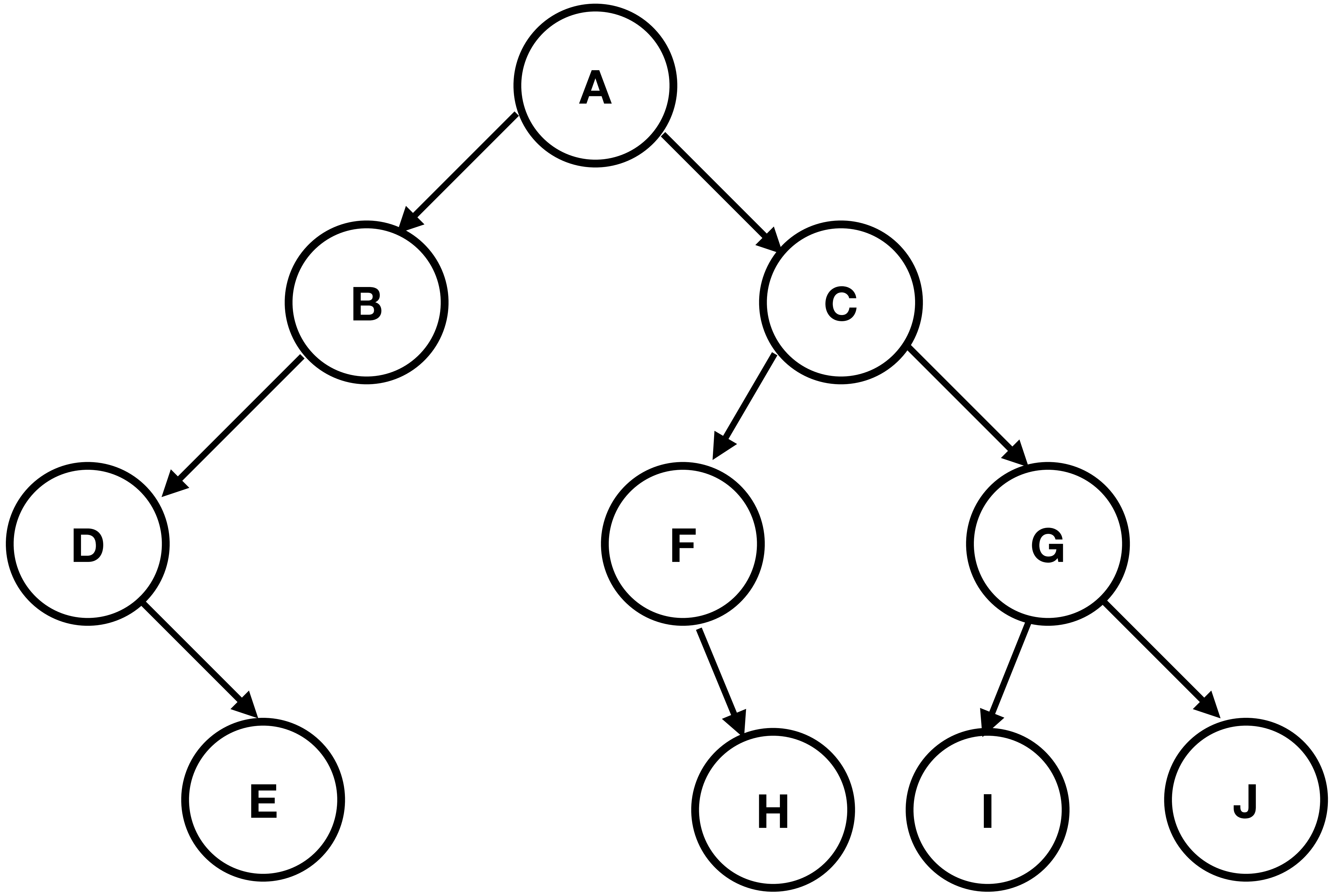
If the tree is not empty
- **postOrder(left subtree)**
- **postOrder(right subtree)**
- **visit(Root)**

Binary Trees

Preorder: A, B, D, E, C, F, H, G, I, J

Inorder: D, E, B, A, F, H, C, I, G, J

Postorder: E, D, B, H, F, I, J, G, C, A

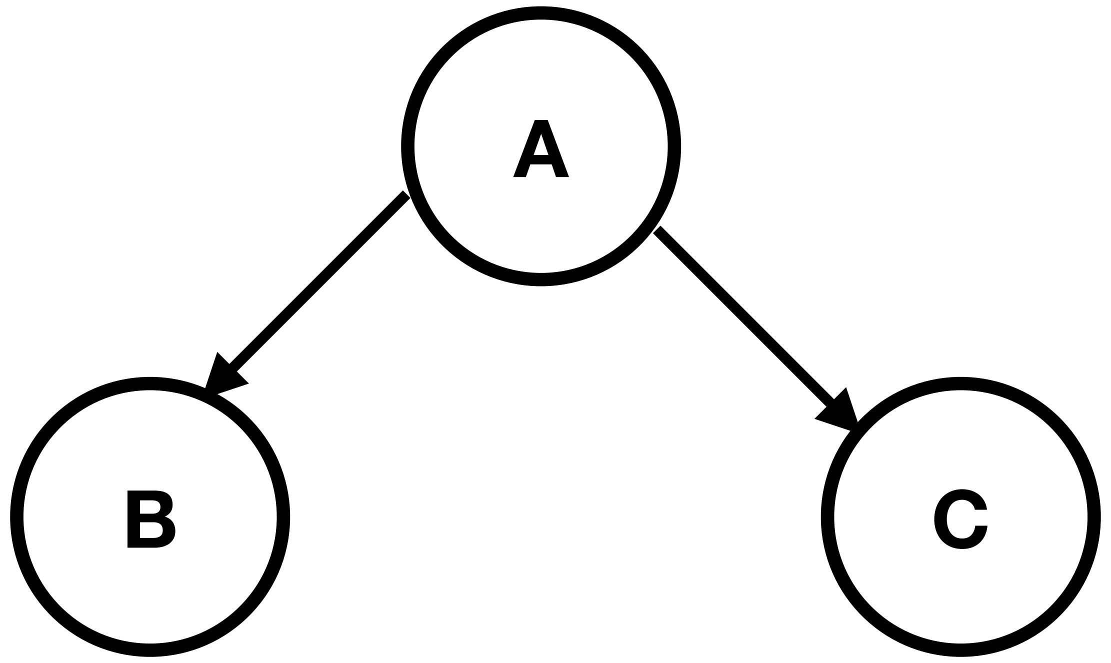


Binary Trees

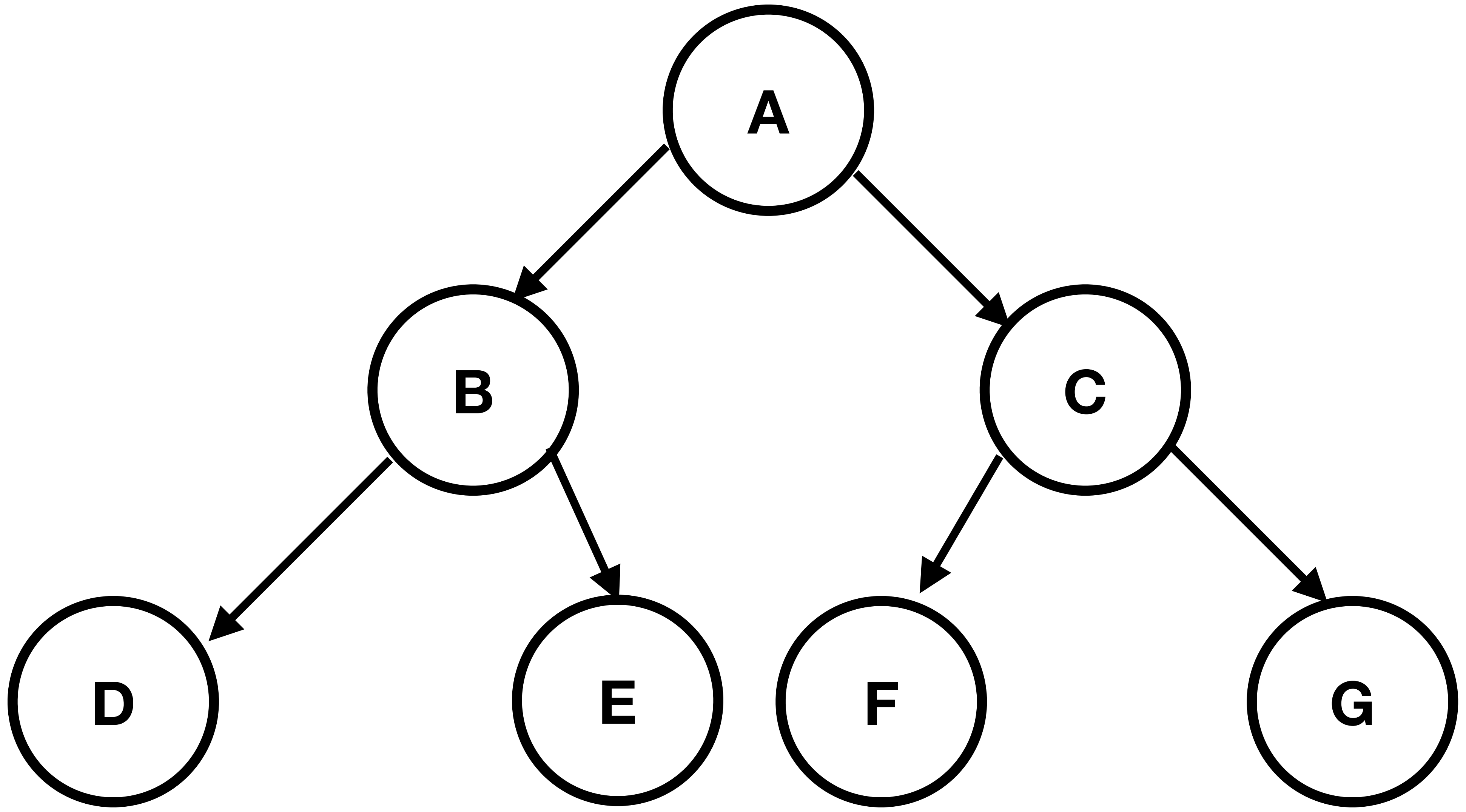
Preorder: A, B, C

Inorder: B, A, C

Postorder: B, C, A



Binary Trees

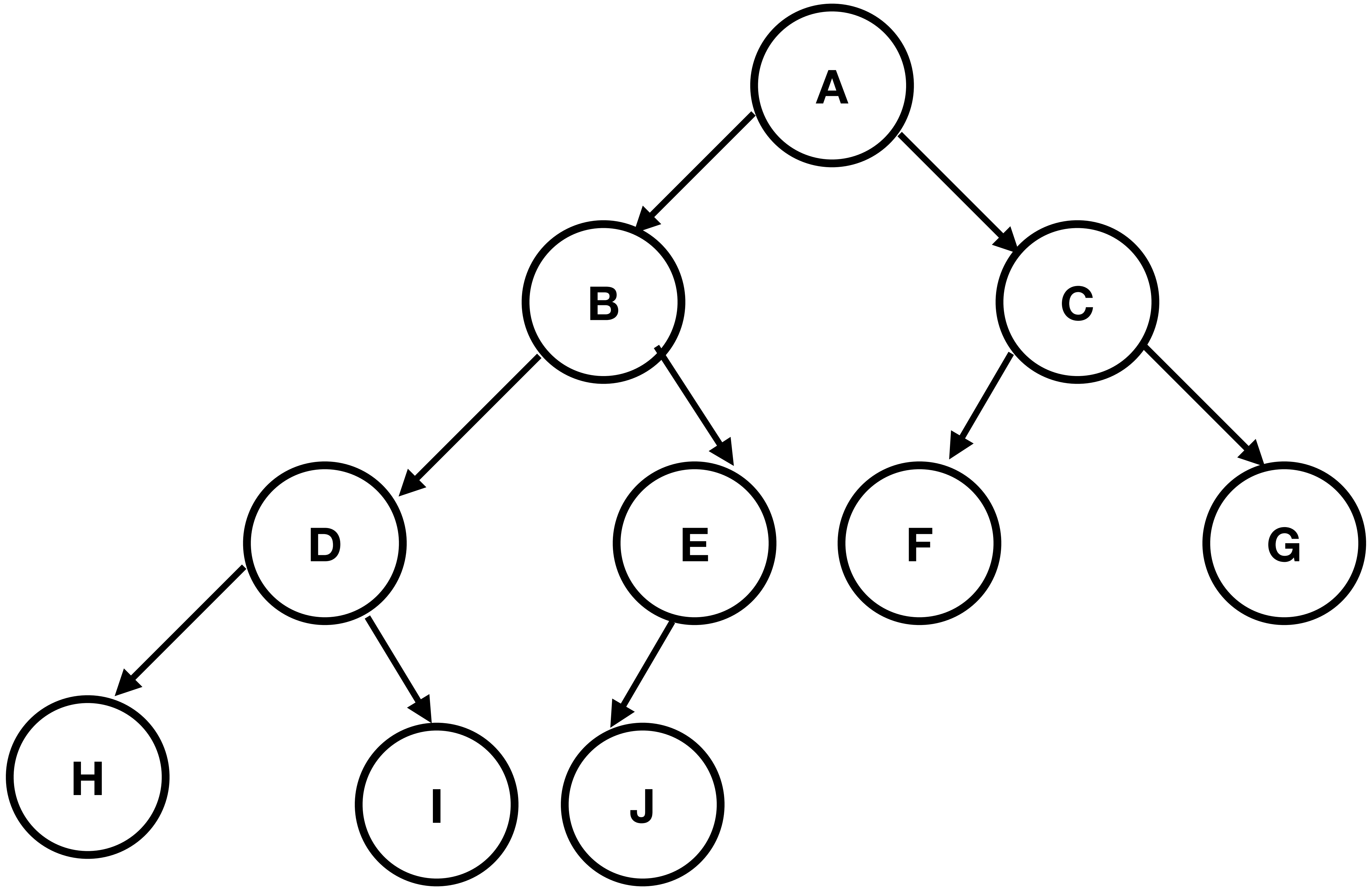


Full Binary Tree: A binary tree where every level has as many nodes as possible (i.e. every level is full).

Number of nodes in a full binary tree: $2^{h+1}-1$ where h is the height of the tree

Maximum number of nodes at depth n: 2^n

Binary Trees



A **complete binary tree** is a binary tree in which every level but the deepest is full and the deepest level is either full or the nodes are as far left as possible.