

Problem Description

La Crosse weather station tries to predict whether or not it will snow in La Crosse on a given day. It uses two factors - whether it is sunny or cloudy on that day and whether the current temperature is above or below the freezing point. The weather station uses a temperature sensor to determine whether the current temperature is above or below the freezing point. Any weather prediction by the station depends on the assumption that the temperature sensor is working correctly. If it is sunny and the temperature is above the freezing point, then the chances of snow falling on that day are LOWER. When it is cloudy, if the temperature is below freezing point, the chances of snow falling on that day is HIGHER. Currently, it is cloudy in La Crosse and the temperature sensor indicates that the temperature is below the freezing point. The station predicted that the chances of snow falling in La Crosse today are HIGHER. Justify, under what condition(s), the prediction would be correct.

Hint: You may have to introduce one or more facts to complete the derivation process.

Let

p - it is sunny

q - the temperature is above the freezing point

r - the chances of snow fall in La Crosse is HIGHER

s - the temperature sensor is working correctly

Rewrite the statements into propositions:

$$p \wedge q \Rightarrow \neg r \quad (1)$$

$$\neg p \Rightarrow (\neg q \Rightarrow r) \quad (2)$$

$$\neg p \quad (3)$$

$$s \Rightarrow \neg q \quad (4)$$

Using (3), **(2)** and Modus Ponens, derive

$$\neg q \Rightarrow r \quad (5)$$

In order to proceed further, we need to have the fact

$$s \quad (6)$$

Using (4), (6) and Modus Ponens, derive

$$\neg q \quad (7)$$

Using (7), (5) and Modus Ponens, derive

$$r$$

Therefore the prediction that ‘the chances of snow fall in La Crosse is HIGHER is correct’ if the temperature sensor is working correctly.