Show that, using the rules for conditional statements, the following two fragments of code are equivalent:

```java
if (x >= 0) {
    if (x >= 0 && y < MAX) {
        System.out.print (''1'');
    }
    else
        System.out.print (''2'');
}
```

For simplicity, we write only the hypotheses in the rules ignoring the bottom portion. This simplification is mainly because the bottom portion in the rule actually shows the code.

Let us use the following symbols in the code:
- \( P \) - the precondition for the fragment of code (same for both)
- \( Q \) - the post-condition for the fragment of code (same for both)
- \( P' \) - in code fragment 2, the post-condition of the first \texttt{if} block which is the same as the precondition of the second \texttt{if} block.
- \( C_1 \) - the condition \( x \geq 0 \)
- \( C_2 \) - the condition \( y < \text{MAX} \)
- \( S_1 \) - the statement \texttt{System.out.println ('}1')\texttt{)}
- \( S_2 \) - the statement \texttt{System.out.println ('}2')\texttt{)}

For the first code fragment,
\[
\begin{align*}
\{P \land C_1\} & \quad \text{inner block} \quad \{Q\} \\
\{P \land \neg C_1\} & \quad S_2 \quad \{Q\}
\end{align*}
\]

Expanding the inner block, this will become
\[
\begin{align*}
\{P \land C_1\} & \quad [(P \land C_1) \land C_2] \quad S_1 \quad \{Q\} \\
& \quad \quad P \land C_1 \land \neg C_2 \Rightarrow Q \quad \{Q\}
\end{align*}
\]

\[
\{P \land \neg C_1\} S_2 \quad \{Q\}
\]

This can be simplified as
\[
\begin{align*}
\{P \land C_1 \land C_2\} & \quad S_1 \quad \{Q\} \quad (1) \\
P \land C_1 \land \neg C_2 \Rightarrow Q & \quad (2) \\
\{P \land \neg C_1\} & \quad S_2 \quad \{Q\} \quad (3)
\end{align*}
\]

For the second code fragment,
\[
\begin{align*}
\{P \land C_1 \land C_2\} & \quad S_1 \quad \{P'\} \\
P \land \neg (C_1 \land C_2) & \Rightarrow P' \quad (4) \\
\{P' \land \neg C_1\} & \quad S_2 \quad \{Q\} \quad (5) \\
P' \land C_1 & \Rightarrow Q \quad (6)
\end{align*}
\]
In order to show that the two code fragments are equivalent, we discuss the four possible situations based on the two conditions and analyze it case by case.

**Case 1**: $C_1$ and $C_2$ are both true.
By equation (1), statement $S_1$ will be executed resulting in post-condition $Q$.
By equation (5), statement $S_1$ will be executed resulting in post-condition $P'$.
By equation (7), post-condition $Q$ will be reached.

**Case 2**: $C_1$ and $C_2$ are both false.
By equation (3), statement $S_2$ will be executed resulting in post-condition $Q$.
By equation (5), the post-condition $P'$ will be reached. Following equation (6), statement $S_2$ will be executed resulting in post-condition $Q$.

**Case 3**: $C_1$ is true and $C_2$ is false.
By equation (2), no statement will be executed and post-condition $Q$ will be reached.
By equation (5), post-condition $P'$ will be arrived at first. Then by using equation (7), post-condition $Q$ will be reached without executing any statement.

**Case 4**: $C_2$ is true and $C_1$ is false.
By equation (3), $S_2$ will be executed resulting in post-condition $Q$.
By equation (5), post-condition $P'$ will be arrived at first. Then using equation (6), statement $S_2$ will be executed resulting in post-condition $Q$.

We have thus proved that both code fragments result in the same post-condition for the same precondition and the conditions in the if statements, and hence they are equivalent.