Show that, using the rules for conditional statements, the following two fragments of code are equivalent:

```
if (x >= 0) {
   if (y < MAX) {
      System.out.print (‘‘1’’);
   }
}
else
   System.out.print (‘‘2’’);
```

```
if (x >= 0 && y < MAX) {
   System.out.print (‘‘1’’);
}
if (!(x >= 0)) {
   System.out.print (‘‘2’’);
}
```

For simplicity, we write only the hypotheses in the rules ignoring the bottom portion. This simplification is mainly because the bottom portion in the rule actually shows the code.

Let us use the following symbols in the code:
P - the precondition for the fragment of code (same for both)
Q - the post-condition for the fragment of code (same for both)
P' - in code fragment 2, the post-condition of the first **if** block which is the same as the precondition of the second **if** block.
C_1 - the condition $x \geq 0$
C_2 - the condition $y < MAX$
S_1 - the statement `System.out.println (‘‘1’’)`
S_2 - the statement `System.out.println (‘‘2’’)`

For the first code fragment,
$\{P \wedge C_1\}$ inner block $\{Q\}$
$\{P \wedge \neg\, C_1\} S_2 \{Q\}$
Expanding the inner block, this will become
$\{P \wedge C_1\} \quad [\{(P \wedge C_1) \wedge C_2\}\ S_1\ \{Q\}$
$P \wedge C_1 \wedge \neg\, C_2 \Rightarrow Q] \quad \{Q\}$
$\{P \wedge \neg\, C_1\} S_2\ \{Q\}$
This can be simplified as

$$\{P \wedge C_1 \wedge C_2\}\ S_1\ \{Q\} \tag{1}$$
$$P \wedge C_1 \wedge \neg\, C_2 \Rightarrow Q \tag{2}$$
$$\{P \wedge \neg\, C_1\}\ S_2 \{Q\} \tag{3}$$

For the second code fragment,

$$\{P \wedge C_1 \wedge C_2\}\ S_1\ \{P'\} \tag{4}$$
$$P \wedge \neg\, (C_1 \wedge C_2) \Rightarrow P' \tag{5}$$
$$\{P' \wedge \neg\, C_1\}\ S_2\ \{Q\} \tag{6}$$
$$P' \wedge C_1 \Rightarrow Q \tag{7}$$

In order to show that the two code fragments are equivalent, we discuss the four possible situations based on the two conditions and analyze it case by case.

Case 1: C_1 and C_2 are both true.
By equation (1), statement S_1 will be executed resulting in post-condition Q.
By equation (5), statement S_1 will be executed resulting in post-condition P'.
By equation (7), post-condition Q will be reached.

Case 2: C_1 and C_2 are both false.
By equation (3), statement S_2 will be executed resulting in post-condition Q.
By equation (5), the post-condition P' will be reached. Following equation (6), statement S_2 will be executed resulting in post-condition Q.

Case 3: C_1 is true and C_2 is false.
By equation (2), no statement will be executed and post-condition Q will be reached.
By equation (5), post-condition P' will be arrived at first. Then by using equation (7), post-condition Q will be reached without executing any statement.

Case 4: C_2 is true and C_1 is false.
By equation (3), S_2 will be executed resulting in post-condition Q.
By equation (5), post-condition P' will be arrived at first. Then using equation (6), statement S_2 will be executed resulting in post-condition Q.

We have thus proved that both code fragments result in the same post-condition for the same precondition and the conditions in the **if** statements, and hence they are equivalent.