Problem Description

People who got injured in road accidents are taken to hospital unless the injuries are not serious. A hospital provides adequate treatment for injured people only if they have proper health insurance. However, people who work at the hospital do not take health insurance. The cost of treatment is higher for those who do not work in the hospital. John Murphy involved in a car accident and had serious injuries. He was given adequate treatment in a hospital and his cost of treatment was lower. Derive a contradiction from the given facts.

Rewrite them in formal statements:

$$\forall\, p : Person \bullet seriouslyInjured(p) \Rightarrow takenToHospital(p) \tag{1}$$
$$\forall\, p : Person \bullet provideTreatment(p) \land takenToHospital(p) \Rightarrow hasHealthInsurance(p) \tag{2}$$
$$\forall\, p : Person \bullet workInHospital(p) \Rightarrow \neg\, hasHealthInsurance(p) \tag{3}$$
$$\forall\, p : Person \bullet \neg\, workInHospital(p) \Rightarrow costOfTreatmentHigher(p) \tag{4}$$
$$\exists\, p : Person \bullet p = JohnMurphy \land seriouslyInjured(p) \land provideTreatment(p) \land$$
$$\neg\, costOfTreatmentHigher(p) \tag{5}$$

Applying One-point-rule to (5), derive
$$seriouslyInjured(JohnMurphy) \land providedTreatment(JohnMurphy) \land$$
$$\neg\, costOfTreatmentHigher(JohnMurphy) \tag{6}$$
Using Conjunctive simplification on 6, derive
$$seriouslyInjured(JohnMurphy) \tag{7}$$
$$providedTreatment(JohnMurphy) \tag{8}$$
$$\neg\, costOfTreatmentHigher(JohnMurphy) \tag{9}$$
Using (1), (7) and Modus Ponens, derive
$$takenToHospital(JohnMurphy) \tag{10}$$
Using (8), (10) and Conjunctive addition, derive
$$takenToHospital(JohnMurphy) \land providedTreatment(JohnMurphy) \tag{11}$$
Using (11), (2) and Modus Ponens, derive
$$hasHealthInsurance(JohnMurphy) \tag{12}$$
Using (9), (4) and Modus Tollens, derive
$$\neg\, (\neg\, workInHospital(JohnMurphy)) \tag{13}$$
Using (13) and Double Negation, derive
$$workInHospital(JohnMurphy) \tag{14}$$
Using (14), (3) and Modus Ponens, derive
$$\neg\, hasHealthInsurance(JohnMurphy) \tag{15}$$

(12) and (15) are contradictory to each other because (12) indicates John Murphy has health insurance but (15) indicates he does not have health insurance.