Genetic Algorithms

Multi-objective Optimization

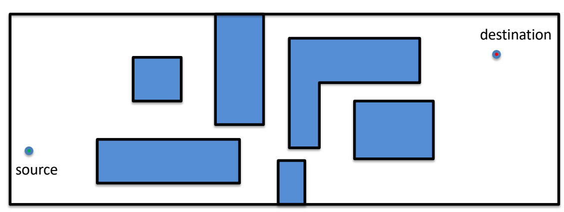
Multi-objective Evolutionary Algorithms

- Multi-objective optimization problems (MOPs)
  - Examples
  - Domination
  - Pareto optimality
  - Practical example
- EC approaches
  - Preference-based
  - Ideal
- Preserving diversity

Multi-Objective Problems (MOPs)

- Wide range of problems can be categorised by the presence of a number of *n* possibly conflicting objectives:
  - robotic path planning:
  - buying a car: speed vs. price vs. reliability
  - engineering design: lightness vs. strength
- Solving an MOP presents two problems:
  - finding set of good solutions
  - choice of best for particular application

Multi-objective problems
Example: Path planning



Goal: find a shortest, obstacle-avoiding path from source to destination.

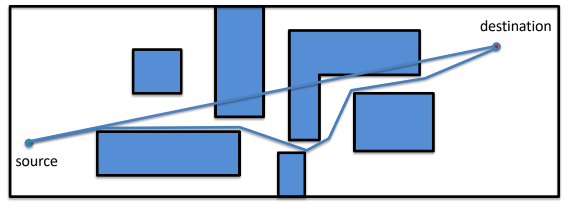
Multi-objective problems
Example: Path planning

- What are the objectives?
  - Path length (minimize)
  - Obstacle collisions (minimize)

- Any others?
  - Number of waypoints (minimize)
  - Smoothness (minimize/maximize – depends on definition)
  - Intermediate destinations?

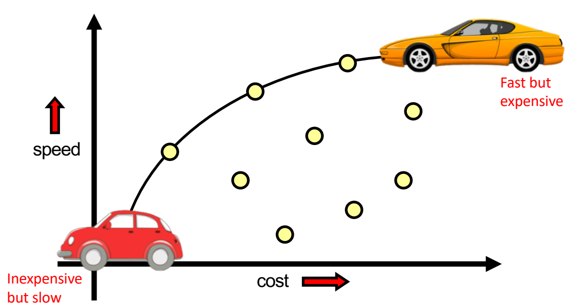
Multi-objective problems
Example: Path planning

Conflicting objectives:



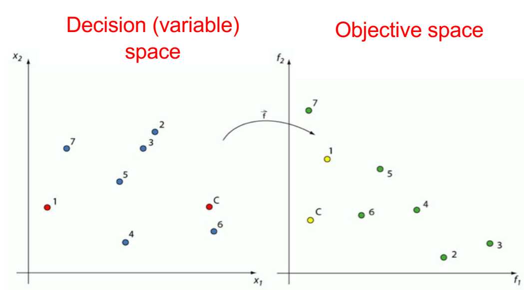
Optimal for obstacle collisions ution?

Multi-objective problems
Example: Buying a car



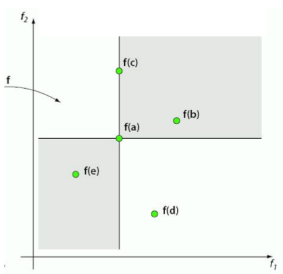
Which is a better solution?

Multi-objective Optimization Problems
Two spaces



Multi-objective Optimization Problems
Comparing Solutions

Objective space

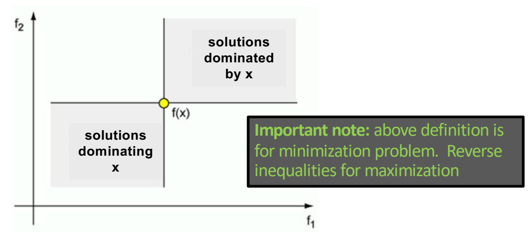


- Optimisation task: Minimize both f_1 and f_2
- Then:
  a is better than b
  a is better than c
  a is worse than e
  a and d are incomparable

Multi-objective Optimization Problems
The Dominance relation

- Solution X dominates solution Y, ($X \preceq Y$), if:
  - X is no worse than Y in every objective
  - X is better than Y in at least one objective

$\forall i \in \{1, \dots, n\}\ x_i \le y_i$ and $\exists i \in \{1, \dots, n\}\ x_i < y_i$



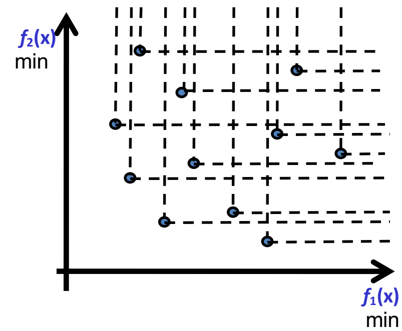
Multi-objective Optimization Problems
Origins of Pareto optimization

- Vilfredo Pareto (1848-1923) was an Italian economist, political scientist and philosopher
- For much of his life he was a political economist at the University of Lausanne (Switzerland)
- *Manual of Political Economy* (1906): described equilibrium for problems consisting of a system of objectives and constraints
- Pareto optimality(economics): an economy is is functioning optimally when no one's position can be improved without someone else's position being made worse

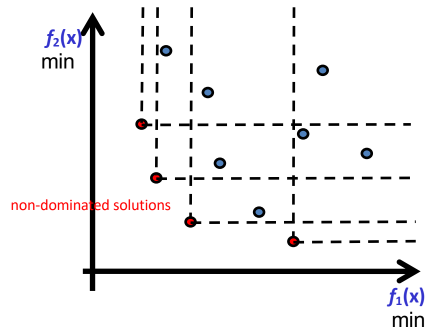
Multi-objective Optimization Problems
Pareto optimality

- Solution x is non-dominated among a set of solutions Q if no solution from Q dominates x
- A set of non-dominated solutions from the entire feasible solution space is the Pareto-optimal set, its members Pareto-optimal solutions
- Pareto-optimal front: an image of the Pareto-optimal set in the objective space

Multi-objective Optimization Problems
Illustration of the concepts

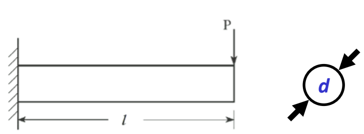


Multi-objective Optimization Problems
Illustration of the concepts



Practical Example:
The beam design problem

Minimize weight and deflection of a beam (Deb, 2001):



Practical Example:
The beam design problem – Formal Definition

- Minimize $f_1(d,l) = \rho \dfrac{\pi d^2}{4} l$ (beam weight)
- minimize $f_2(d,l) = \delta = \dfrac{64 P l^3}{3 E \pi d^4}$ (beam deflection)
- subject to $0.01 \text{ m} \le d \le 0.05 \text{ m}$

$0.2 \text{ m} \le l \le 1.0 \text{ m}$

$\sigma_{\max} = \dfrac{32 P l}{\pi d^3} \le S_y$ (maximum stress)

$\delta \le \delta_{\max}$

where $\rho = 7800 \text{ kg/m}^3,\ P = 2 \text{ kN}$

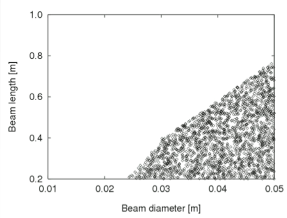
$E = 207 \text{ GPa}$

$S_y = 300 \text{ MPa},\ \delta_{\max} = 0.005 \text{ m}$

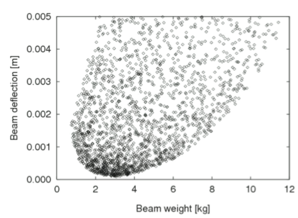
Practical Example: The beam design problem

Feasible Solutions:

Decision (variable) space

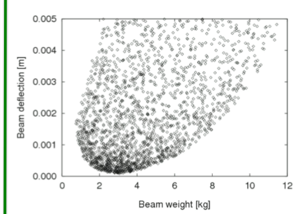


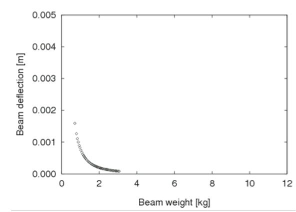
Objective space



Practical Example: The beam design problem

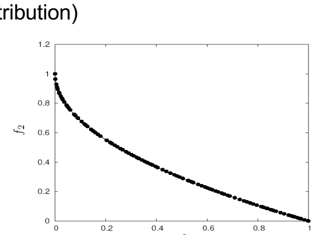
Goal: Finding non-dominated solutions:





Goal of multi-objective optimizers

- Find a set of non-dominated solutions (approximation of the Pareto-optimal front) following the criteria of:
  - convergence (as close as possible to the Pareto-optimal front)
  - diversity (spread, distribution)



Single vs. Multi-objective Optimization

Characteristic	**Singleobjective optimisation**	**Multiobjective optimisation**
Number of objectives	one	more than one
Spaces	single	two: decision (variable) space, objective space
Comparison of candidate solutions	*x* is better than *y*	*x* dominates *y*
Result	one (or several equally good) solution(s)	Pareto-optimal set
Algorithm goals	convergence	convergence, diversity

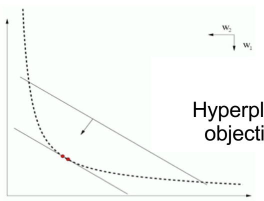
Multi-objective optimization
Two approaches

- Preference-based:
  traditional, using single objective optimisation methods
- Ideal:
  possible with novel multiobjective optimisation techniques, enabling better insight into the problem

Multi-objective optimization
Preference-based approach

- Given a multiobjective optimisation problem,
- use higher-level information on importance of objectives
- to transform the problem into a singleobjective one,
- then solve it with a single objective optimization method
- to obtain a particular trade-off solution.

Multi-objective optimization
Preference-based approach



Hyperplanes in the objective space!

Modified problem: $F(\mathrm{x}) = \sum_{m=1}^{M} w_m f_m(\mathrm{x}), \quad w_m \in [0,1], \quad \sum_{m=1}^{M} w_m = 1$

The weighted sum scalarizes the objective vector: we no have a single-objective problem

Multi-objective optimization
Ideal approach

- Given a multiobjective optimization problem,
- solve it with a multi-objective optimization method
- to find multiple trade-off solutions,
- and then use higher-level information
- to obtain a particular trade-off solution.

EC approach to multi-objective optimization: Advantages

- Population-based nature of search means you can *simultaneously* search for set of points approximating Pareto front
- Can return a set of trade-off solutions (approximation set) in a single run
- Don't have to make guesses about which combinations of weights might be useful
- Makes no assumptions about shape of Pareto front - can be convex / discontinuous etc.

EC approach to multi-objective optimization: Requirements

- Way of assigning fitness,
  - usually based on dominance
- Preservation of diverse set of points
  - similarities to multi-modal problems
- Remembering all the non-dominated points you have seen
  - usually using elitism or an archive

EC approach: Fitness assignment options

- Could use aggregating approach and change weights during evolution
  - no guarantees
- Different parts of population use different criteria
  - e.g. VEGA, but no guarantee of diversity
- Dominance
  - ranking or depth based
  - fitness related to whole population
  - Question: how to rank non-comparable solutions?

EC approach: Diversity maintenance

- Usually done by niching techniques such as:
  - fitness sharing
  - adding amount to fitness based on inverse distance to nearest neighbour (minimisation)
  - (adaptively) dividing search space into boxes and counting occupancy
- All rely on some distance metric in genotype / phenotype space

EC approach: Remembering good solutions

- Could just use elitist algorithm
  - e.g. ($\mu + \lambda$) replacement
- Maintain an archive of non-dominated solutions
  - some algorithms use this as second population that can be in recombination etc.
  - others divide archive into regions too, e.g. PAES

Multi-objective optimization Problem Summary

- MO problems occur very frequently
- EAs are very good at solving MO problems
- MOEAs are one of the most successful EC subareas